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CAPILLARY WAVES ON THE SURFACE OF A THIN LAYER OF A CHARGED CONDUCTING LIQUID

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It is shown that if a thin liquid layer is brought into the state of steady motion, the surface tension decreases efficiently for waves propagating along the flow or in opposition to it and the conditions of occurrence of the instability of a charged liquid surface are made substantially easier.

If an electric charge is supplied to a conducting liquid, the liquid surface begins to fail once a certain threshold density of the charge is exceeded: the electrostatic instability of the liquid is realized [1]. This phenomenon was described for the first time by Rayleigh as far back as 1882 as applied to spherical droplets and by Ya. I. Frenkel' and L. Tonks in 1935 for the plane surface of a massive liquid conductor. In recent times, the phenomenon has comprehensively been studied [2–5] for different geometries, with allowance for the relaxation processes in the liquid, and for the cases of parametric excitation of instabilities.

A situation of practical interest exists which has not yet been studied, namely: in all the investigations it was assumed that, in the initial state (before the development of any instabilities), the liquid is stationary. However horizontal flows exist in the liquid in the most general case. If a massive liquid of infinite depth is to be found in shear flow with a constant velocity, then, with allowance for the relativity of motion, the conditions of occurrence of instability remain the same as for a stationary liquid. If there is a horizontal stream of a liquid of finite thickness, the velocity in this liquid is different at different depths and becomes zero at the bottom, and it cannot be eliminated by any conversion to another reference system. This motion must, apparently, exert an influence on the spectrum of long waves propagating in parallel to the direction of the flow. For such waves whose length is comparable to or larger than the thickness of the liquid layer, the conditions of occurrence of surface instability in the electric field must also change. This work seeks to study surface motions of a conducting liquid of finite depth in the state of a steady flow.

Let us use the following coordinate system: the x axis is directed along the flow, the y axis is directed normally to the surface upward, and the origin of coordinates is located on an undisturbed surface; the bottom corresponds to the coordinate $y = -h$ (h is the flow depth). We consider the steady undisturbed liquid motion which is described by the continuity and Navier–Stokes equations. These equations with one nonzero component of the velocity $V_x(y)$ are rewritten in the form

$$\frac{\partial V_x}{\partial x} = 0 \, , \quad \frac{\partial p}{\partial y} = 0 \, , \quad -\frac{\partial p}{\partial x} + \eta \frac{\partial^2 V_x}{\partial y^2} = 0$$

(it has been taken into account that $(\mathbf{V}\nabla)\mathbf{V} \equiv 0$). Specifying the boundary conditions

$$V_x \big|_{y=-h} = 0 \, , \quad \left. \frac{\partial V_x}{\partial y} \right|_{y=0} = 0 \, ,$$

we obtain

$$p = \frac{\partial p}{\partial x} x \, , \quad V_x = -\frac{1}{2\eta} \frac{\partial p}{\partial x} (h^2 - y^2) \, ,$$

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or, if the velocity $\mathbf{u}$ of the liquid surface is introduced, we have

$$\mathbf{V} = \mathbf{u}\left(1 - \frac{y^2}{h^2}\right). \tag{1}$$

Let us consider the wave motion of the liquid surface. The most important case corresponds to the waves in directions which are in parallel to the flow. We restrict ourselves to this case in this work. We employ the assumption of smallness of the amplitude of vibrations (in relation to the wavelength and the liquid depth) [1]. As has been indicated above, the influence of the liquid flow on wave motion should be expected in the longwave limit. It is precisely this case that will be considered. We denote the deviation of the surface from equilibrium by $\xi(x, t)$. The change in the horizontal component of the velocity V_x will be quadratic in ξ, while the vertical component of the velocity V_y will change in proportion to ξ. As a result of linearization, the Navier–Stokes equation (with the use of (1)) is transformed as

$$\frac{\partial V_y}{\partial t} + u\left(1 - \frac{y^2}{h^2}\right)\frac{\partial V_y}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu \Delta V_y . \tag{2}$$

We are interested in the case where the role of the horizontal flow of the liquid is the most important; therefore, we take $uh > \nu$ and $uk > \omega$, where ω and k are the frequency and the wave number for surface motion (the latter inequality is checked upon completion of computations). Then Eq. (2) yields a simple relationship of the pressure and V_y:

$$\frac{\partial p}{\partial y} = -\rho u\left(1 - \frac{y^2}{h^2}\right)\frac{\partial V_y}{\partial x} . \tag{3}$$

In the case of a constant horizontal flow on the liquid surface the relation

$$V_y \big|_{y=0} = u\frac{\partial \xi}{\partial x} . \tag{4}$$

holds. Computation is made much easier if we assume that such a relation holds throughout the depth of the liquid (with a natural correction of the horizontal velocity component):

$$V_y = u\left(1 - \frac{y^2}{h^2}\right)\frac{\partial \xi}{\partial x} . \tag{5}$$

Substitution of (5) into (3) and integration for y going from 0 to h yield the expression for p on the liquid surface:

$$p = -\frac{8}{15}\rho u^2 h \frac{\partial^2 \xi}{\partial x^2} . \tag{6}$$

The last relation has a simple physical meaning. Indeed, if the surface curvature is written in the form [6] $R^{-1} = -\frac{\partial^2 \xi}{\partial x^2}$ (where R is the radius of curvature and the surface wave is represented in the form $\xi = \xi \exp(ikx - i\omega t)$), the centripetal acceleration of the spinning mass is written as $a = u^2/R$, and the effective thickness of the layer is written in the form $l = \frac{8}{15}h$, formula (6) can be represented as $p = (\rho l)a$, i.e., p is the force that provides the centripetal acceleration a for the mass (ρl).

Expressing the pressure by the radius of curvature and using the Laplace formula for the capillary pressure $p_c = \alpha/R$, we introduce the effective coefficient of surface tension

$$\alpha_e = \alpha - \frac{8}{15} \rho u^2 h \,, \tag{7}$$

and for the spectrum of capillary waves on the surface of a charged conducting liquid we use the known formula [1]

$$\omega = \sqrt{\frac{k^2}{\rho} (\alpha_e k - 4\pi\sigma_0^2)} \;. \tag{8}$$

Here σ_0 is the surface density of the charges (related to the electric-field strength by the formula $E = 4\pi\sigma_0$).

Let us give the numerical estimates. For mercury (ρ = 13.6 g/cm^3) we have $\alpha_e = 0.4\alpha$ when u = 10 cm/sec and the thickness is h = 0.5 cm and $\alpha_e = 0$ when u = 14 cm/sec.

Thus, the result (7) is fundamental: a decrease in the surface tension leads to a reduction in the threshold values for the electric field in the case of the Frenkel'–Tonks surface instability.

NOTATION

ρ, η, and ν, density and dynamic and kinematic viscosities of the liquid; p and V, pressure and velocity of the flow; ξ, ω, and k, amplitude, frequency, and wave number of the surface wave; α and α_e, ordinary and effective coefficients of surface tension. Subscripts: c, capillary; e, effective.

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